

Ex: Solve $y = p \tan p + \log \cos p$

Ans: We have $y = p \tan p + \log \cos p$ --- (1)

Differentiating (1) with respect to x
and writing p for dy/dx we get,

$$p = [\tan p + p \sec^2 p + \frac{1}{\cos p} (-\sin p)] \frac{dy}{dx}$$

$$\text{or } dx = \sec^2 p dp \text{ --- (2)}$$

Integrating (2) we get

$$x = \tan p + c \text{ --- (3), } c \text{ being an arbitrary constant}$$

(1) & (3) gives the solution in parametric form, p being the parameter.

Ex: Solve $p^3 + p = e^x$

Ans: We have $p^3 + p = e^x$

Taking logarithm of both sides of the given equation we get

$$y = \log\{p(1+p^2)\} = \log p + \log(1+p^2) \text{ --- (1)}$$

Differentiating (1) w.r.t. x we have

$$p = \left(\frac{1}{p} + \frac{2p}{1+p^2} \right) \frac{dp}{dx}$$

$$\text{or } dx = \left(\frac{1}{p^2} + \frac{2}{1+p^2} \right) dp$$

Integrating we get

$$x = -\frac{1}{p} + 2 \tan^{-1} p + c \text{ --- (2), } c \text{ being an arbitrary constant.}$$

(1) & (2) together give the required solution in parametric form, p being the parameter.

Solve the following equations:

$$1) y = \sin p - p \cos p \quad 2) e^{p-y} = p^2 - 1$$

$$3) y = a\sqrt{1+p^2} \quad 4) y = 2p + 3p^2$$

$$5) p - y = \log(p^2 - 1) \quad 6) y = 2p + \sqrt{1+p^2}$$

(f) Equations not containing y

If the given equation does not contain y , and let it be put in the form

$$f(x, p) = 0, \quad p = \frac{dy}{dx} \quad \dots (1)$$

Then the equation is either solvable for p or it is solvable for x . Hence using the methods as in the previous class, the complete primitive is determined.

Solve the following equations:

$$7) p^2 - 2xp + 1 = 0 \quad 8) 4xp^2 = (3x - a)^2$$

$$9) x(1+p^2) = 1 \quad 10) x + \left(\frac{p}{\sqrt{1+p^2}} \right) = a$$